Could synchrotron light sources benefit from the experience at CERN with beams split in horizontal phase space?

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This article reviews the recent activities carried out at CERN in terms of transverse beam splitting with proton beams. Thanks to intense experimental and theoretical activities, the recently-proposed approach of splitting beams in the horizontal phase space using stable islands moved out of the test stage and it became a mature technique of beam manipulation to perform multi-turn extraction. It is therefore timely to address the question whether stable islands can be applied to other branches of accelerator physics, in view of bringing new means of improving ring's performance. This is the case of synchrotron light machines, where the use of stable islands could lead to new developments, and in this paper options are presented and discussed.

Keywords: Non-linear beam dynamics, stable islands, radiation damping, quantum excitation

1. Introduction

The efforts to find a suitable replacement of the Continuous Transfer (CT) extraction mode[1,2], which has been the technique used to transfer the 14 GeV/c proton beams from the CERN Proton Synchrotron (PS) to the Super Proton Synchrotron (SPS) for the CERN fixed-target physics programme, converged to the proposal of a novel beam manipulation named Multi-Turn Extraction (MTE)[3]. This technique is based on transverse beam splitting induced by crossing a stable resonance in the horizontal phase space and solves the issue related to the unavoidable beam losses of the CT extraction[4]. In the case of MTE, the stable fourth-order resonance is used, which generates four beamlets around a central core. In general, any resonance (stable or unstable) can be considered to design a multi-turn extraction[5].

MTE has been studied both numerically and experimentally[6–12]. Subsequently, a detailed implementation proposal was made[13,14], which envisaged the implementation of several changes to the PS ring layout during the various shut-down periods from 2004 to 2008. The years following the end of the installation phase were dedicated to hardware and beam commissioning[15–17]. A second phase was launched in 2012 to study, design, and install additional hardware to mitigate beam losses on the extraction septum due to the de-bunched beam structure and the long rise time of the extraction kickers[18–22]. The installation of the newly-developed hardware was accomplished during the Long Shut-down 1 (February 2013 - May 2014), and finally, MTE has been successfully commissioned and put in operation by the second half of 2015[23–25].

Even if the novel beam manipulation, based on key concepts of non-linear beam dynamics, was originally linked with a specific need, i.e., reducing the extraction losses for a special extraction mode, it can be generalised to provide novel tools for improving the performance of proton rings. For instance, the splitting process can be time-reversed thus generating a Multi-Turn Injection (MTI) gymnastics[26]. The interest of MTI is that it can provide a mitigation to space charge effects by means of a shaping of the transverse beam distribution (see Ref.[27] for an account of preliminary results on this topic). Additionally, stable islands can provide a second closed orbit with different properties, in terms of optical parameters, with respect to the standard closed orbit. All this opens new options and scenarios for the use of a circular particle accelerator. In fact, two beams can be stored simultaneously in order to exploit the possibility of featuring different optical properties. Alternatively, a single beam can be moved selectively between the central orbit and the stable islands in order to change its properties according to the user's needs.

Following the positive results of these studies, it was natural to move forward and to investigate the possibility to extend these techniques to new domains, and that of lepton machines and synchrotron light sources is certainly the most promising one. The main challenge is the impact of radiation damping and quantum excitation, the two key differences between lepton and proton beam dynamics, on the very existence of stable islands and on their properties. The analysis of the properties of stable islands for lepton rings is the focus of this paper, including also the possibility of extending the known formulas governing the motion of leptons around the standard closed orbit, i.e., that at the centre of phase space, to the case of leptons inside stable islands, and the conditions that make leptons remaining stably inside the islands.

It is worth noting that possible applications of stable islands to lepton rings have been studied and experimentally tested with users at BESSY II. An overview on the status of these activities can be found in[28] and references therein, as well as in[29]. However, it should be stressed that in BESSY II the population of stable islands is achieved by a continuous beam excitation, generated by means of a transverse kicker, which represents a different approach with respect to the scenarios considered in this paper.

The plan of the paper is the following: in section 2, the applications of stable islands for proton beams are reviewed, covering both the dynamic 2.1 and the static 2.2 case. In section 3, the case of lepton beams is considered discussing the possible uses of stable islands, whereas in section 4 the results of numerical simulations are presented, discussing the computation of optical parameters and equilibrium emittances using analytical 4.1 or tracking 4.2 tools. Finally, conclusions are drawn in section 5.

2. Stable islands for proton beams

2.1. *Dynamic use of stable islands*

This application has been intensively studied at CERN and used to split transversely a proton beam. The heart of the method is the generation of stable islands by means of magnets generating non-linear fields such as sextupoles and octupoles, combined with a slow change of the transverse machine tune. If the tune change is adiabatic with respect to the betatronic motion[30], then particles can be trapped inside the stable islands that are sweeping through the region of phase space where the beam sits. In this way, a single beam with Gaussian transverse distribution can be split into $N+1$ beamlets, assuming a stable resonance of order N is crossed, with one beamlet moving around the standard closed orbit and the other N around the closed orbit of the stable fixed point at the centre of the islands. Note that the beamlet around the standard closed orbit generates a structure with a length corresponding to that of the ring, while the other beamlets form a single structure closing up after N turns around the ring circumference. As already mentioned, such an approach can be used to either split or merge (by time-reversal of the splitting method) proton beams.

2.2. *Static use of stable islands*

Alternatively, the machine tune and all other settings can be kept constant and in this case the energy-conservation property of the protons' dynamics ensures that the separatrix around the stable islands cannot be crossed, thus preventing any beam splitting. Also this configuration can be used to improve the performance of proton rings. As already mentioned, two closed orbits exist simultaneously in the ring for such a configuration and can be used selectively. The key observation is that the optical parameters of the closed orbit related with the stable islands are different from the nominal ones. In fact, a particle moving on the closed orbit that is at non-zero distance from the origin of phase space experiences the feed-down effects from higher-order magnetic fields, such as, e.g., sextupoles and octupoles, which produce additional quadrupolar components with respect to those stemming from the main quadrupolar magnets. An example of this is shown in Fig. 1, where the evolution of one of the fixed point positions along the circumference of the PS is depicted. Note that the standard vacuum pipe is elliptical in shape with full dimensions 140×70 mm^2. In addition, the optical parameters for the stable islands of the fourth-order resonance are compared against those of the core.

The model used for the numerical computations is made of four copies of the PS ring, in order to restore the appropriate periodicity for the stable islands. The difference in optical parameters in the horizontal plane is clearly visible. In the vertical plane, differences only up to 50 % are observed, which is a sign of the lim-

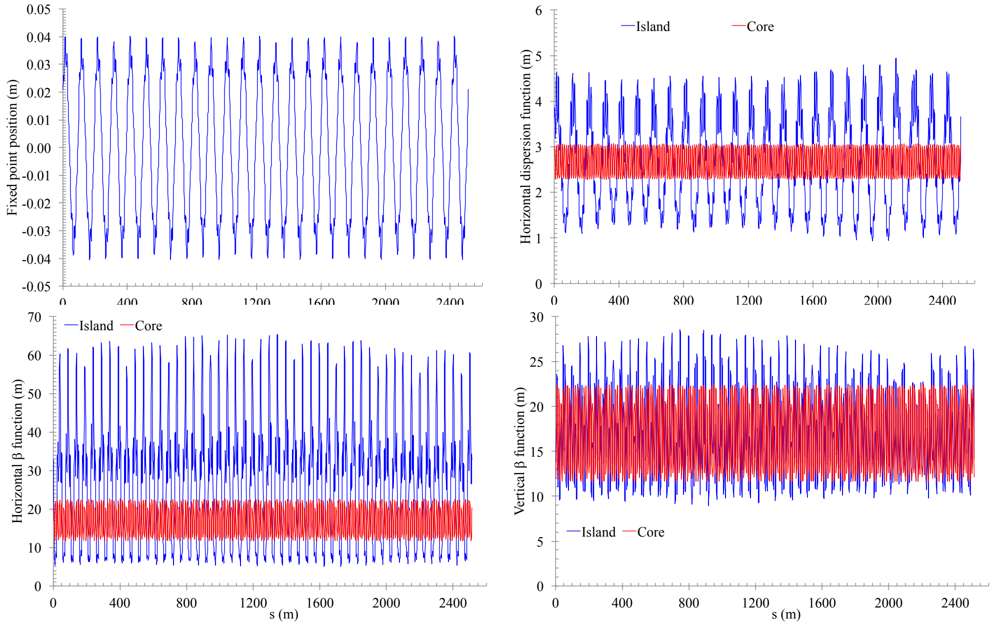

Fig. 1. Evolution of the fixed point position (upper left) along the PS-ring circumference. Also shown is the comparison of optical parameters for stable islands against those of the core for horizontal beta function (lower left), horizontal dispersion (upper right), and vertical beta function (lower right). The differences are clearly visible in the horizontal plane, while smaller changes are observed in the vertical plane.

ited impact of the unavoidable non-linear coupling between horizontal and vertical motion introduced by sextupoles and octupoles. Another interesting observation is that the optical parameters for islands and core can be controlled independently from each other. Although quadrupoles affect both core and islands, non-linear magnets change optical parameters for the islands, only.

Starting from this result, it is possible to move forward to the analysis of the coupling effects between horizontal and longitudinal motion. This is summarised in the value of the momentum compaction α_c that expresses the orbit lengthening as a function of momentum offset. The variation of this parameter is shown in Fig. 2 for the standard closed orbit and that related with the stable islands.

Also in this case, the differences between core and islands are clearly visible. Moreover, it is worth noting that the value of α_c can be made different between the two closed orbits, thus showing the possibility to generate very different optics in the same ring depending on the selected closed orbit.

3. Stable islands for lepton rings

Two main different uses of stable resonant islands may be of great interest to the synchrotron light source community.

First, beam trapped in stable islands would allow users of different communities

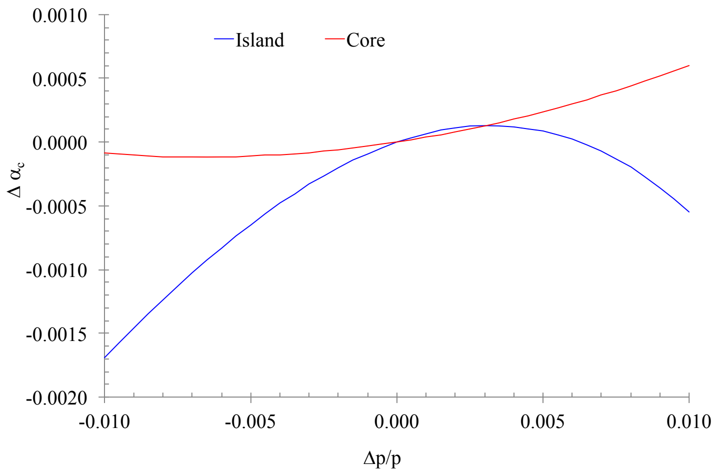

Fig. 2. Dependence of $\Delta\alpha_c$ on momentum offset for islands and core. The different behaviour is clearly visible. Note that $\alpha_{c,\text{islands}} = 3.10 \times 10^{-2}$ while $\alpha_{c,\text{core}} = 1.73 \times 10^{-2}$.

to operate simultaneously without disturbing each other. Without loss of generality, users of modern synchrotron light sources can be split in two main groups. The first requires the smoothest and the highest possible photon flux (and brilliance most of the time), with no interest in the temporal (i.e., the bunch) structure of the electron beam: This community would then prefer to work with a continuous multi-bunch train with the lowest beam current per bunch compatible with the maximum total current. The second group of users needs instead specific temporal structures of the electron beam, being interested in time-resolved, pump-probe, and time-of-flight experiments: This community requires a single or a few-bunch filling pattern with high charge per bunch, with a limited interest in the brilliance and integrated photon flux. To satisfy both, the available beam time needs to be split between uniform filling pattern and single or few-bunch operation modes. More recently, a hybrid scheme was proposed where a large fraction of the ring, say 7/8, is filled with a continuous train of low-charge bunches and an intense single bunch is placed at the centre of the remaining gap. This mode is referred to as *camshaft* and an example taken from the European Synchrotron Radiation Facility (ESRF) is shown in Fig. 3, where a single bunch of 8 mA is placed at the centre of the gap separating a uniform bunch train of 868 bunches totalling 192 mA. With this filling pattern, time-structure users select only the photons generated by the single bunch while gating out those stemming from the multi-bunch train. Flux-driven users instead utilise all photons with no distinction of the source. While being effective, as it allows a sharing of the beam time, this operation mode has some drawbacks. The ungated flux-driven community may suffer from glitches originated by a partial saturation of their detectors when the intense photon pulse, stemming from the single bunch, arrives, the detectors being optimised for the uniform bunch train. Unless a mechanical chopper is used, users of the single bunch are limited

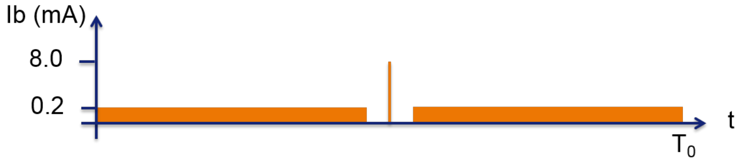

Fig. 3. Schematic view of the *camshaft* filling patter of the ESRF storage ring with 7/8 of the ring is filled with a uniform low-current bunch train, whereas a single, intense bunch is placed at the centre of the gap.

in their temporal resolution by the gap, 1/8 of the revolution time in the case of the ESRF: For this reason, the physics programme schedules weeks of single, 4 and 16 bunches are allocated along the *camshaft* and the uniform filling modes. The limited lifetime of the high-charge single bunch conflicts also with the larger lifetime of the multi-bunch train, since it is customary to have a single top-up procedure for all bunches, usually timed on the total beam current. Time-resolved users hence benefit of the high charge of the single bunch only for a limited amount of time.

Resonant islands offer a way out to allow a better compatibility and sharing of the beam time between these two user communities. The single or few bunches needed for time-resolved measurements can be indeed put into the islands, leaving the multi-bunch train onto the nominal orbit. After displacing their beam lines to receive photons from a off-centred (both in position and angle) source point, time-resolved users would no longer need to gate the photon beam, those generated by the multi-bunch train not entering their front-ends. At the same time, users of the multi-bunch train would no longer be disturbed by the temporary and partial saturation of their detectors due to the intense single bunch. Moreover, an arbitrary number of bunches along the ring can be put into the islands, thus granting a larger flexibility on the number of bunches to be used, and hence on the temporal structure of the photon beam. This flexibility extends to one bunch each N revolution time, if only one of the N islands is populated.

Top-up injection could also be performed, independently on the two types of beam, ensuring the highest possible intensity per bunch into the islands, provided that a dedicated scheme to inject directly into the islands is conceived. More generally, operating multiple beams into islands and core could provide an alternative to canting for users sharing the same straight sections, with no need of the traditional scheme with three dedicated dipole magnets generating a closed-orbit bump.

Another technique to separate transversely individual electron bunches was successfully implemented at the Advanced Photon Light Source (ALS) with a *vertical kick-and-cancel* scheme[31]: The single bunch at the centre of the gap is displaced by a closed bump around the front-end housing the time-resolved users. Fast kickers (< 100 ns) ensure that the main bunch train is not perturbed. However, the possibility of having such fast kickers suitable for high-energy rings remains to be assessed. Mechanical choppers[32] (slotted rotating disks) are also implemented in some laboratories to remove photons from unwanted bunches. This approach, however, is limited by the maximum rotational speed and power deposited onto the plates, hence becoming an expensive solution for high-energy facilities.

Another user community that could benefit from one or few bunches trapped into islands is that making use of short bunches to generate short and possibly coherent synchrotron radiation pulses to study rapidly evolving systems or systems far from equilibrium. Linac-based free electron lasers (FEL) provide the shortest pulses, though their low repetition rate and limited beam stability may prevent the feasibility of some types of experiments in such facilities. Storage-ring-based light sources can provide short bunches (though longer than FEL's) at high repetition rate and with great stability by reducing the momentum compaction close to zero or even to low negative values [33]. So-called *low-alpha modes* have been offered to storage-ring-based users since more than a decade. The poor lifetime and low instability threshold typical of these operational modes generate once again a conflict between users. If short bunches are placed into islands this contrast may be solved, hence allowing a simultaneous operation of two communities.

At BESSY II, superconducting cavities working at 3 and 3.5 times the nominal RF frequency are being designed to provide filling schemes with alternating short and long bunches, with the short ones put into islands of horizontal phase space [34]. Non-linear magnets can be used to the same aim. Because the optics of the islands differ from that of the beam on the standard central orbit, specific non-linear magnet settings can be studied to reduce the momentum compaction of the islands only, leaving that of the core unchanged. This scheme would be less costly than the one based on harmonic cavities and arguably the only viable solution for high-energy facilities. Moreover, the short bunch into the island could be individually selected, whereas with harmonic cavities the number of short and long bunches is fixed.

A second possible application to ring-based light sources is represented by the possibility of topping up a stored beam without the use of a closed injection bump. The latter, no matter how well tuned and corrected is the machine optics, is a source of temporary perturbations to the stored beam in terms of orbit distortion (because of its non-closure, of the presence of sextupoles within the bump, and of the energy change due to the longer path length) and beam blow-up (because the linear optics with the injection bump is not matched and of the orbit distortion along the ring). Operational diagnostic tools, such as beam position monitors in slow-mode acquisition (typically at 10 Hz) and x-ray pinhole cameras or detectors, integrate the beam signal over a time (ms or s) much longer than the typical perturbation (~ 100 μs), making possible to detect macroscopic perturbations, only. By eliminating the injection orbit bump most of these disturbances to the stored beam shall disappear. The second closed orbit generated by the fixed points can be exploited to replace the injection bump. The idea is an extension of the injection scheme proposed for hadron machines in Ref. [35] and is schematically represented in Fig. 4. A non-linear optics is designed to generate N unpopulated stable islands

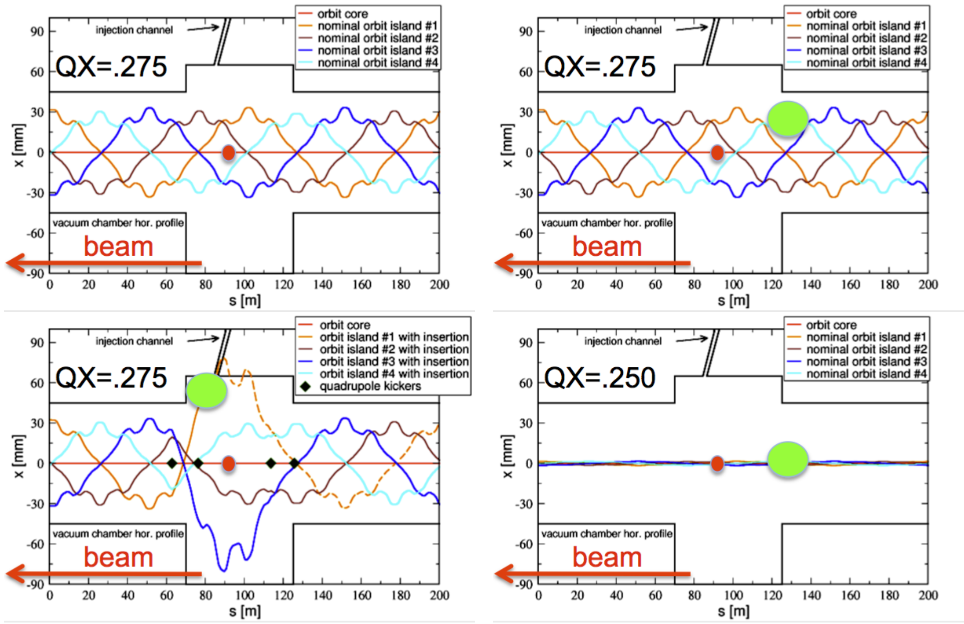

Fig. 4. Schematic view of the proposed top-up leaving the stored beam (red circle) on axis. The injected beam is represented by the green circle and the scheme is based on the creation of 4 stable islands whose orbit is first distorted by a local optics insertion generated by 4 quadrupole kickers and then displaced on axis by moving the tune onto the resonance. The same lattice of the CERN PS of Ref.[35] has been used.

(4 in the upper left figure). Around the injection point, a closed insertion optics is created by quadrupole kickers that displaces locally one of the islands orbit at the exit of the injection septum so to accommodate the incoming beam (lower left figure). Within one or N turns the quadrupole kickers can be switched off (upper right figure) with no risk for the injected beam to intercept the injection septum, because before the N^{th} turn the other orbits will be occupied. The islands are then moved on axis by moving the tune onto the resonance (lower right plot): This can be performed over the injected bunch by using the bunch-by-bunch electronics. A harmonic excitation at a frequency close to the horizontal tune can be imparted to the selected bunch: This excitation acts like an AC dipole used in hadron colliders and hence as an effective quadrupole (see Ref.[36] and references therein). By varying the excitation frequency and amplitude, the island can be moved on axis. Radiation damping would eventually provide the nominal equilibrium emittance. This gymnastics does not displace at all the stored beam, and the only expected perturbation to it may come only from the non-perfect closure of the insertion optics generated by the quadrupole kickers, hence generating a temporary blow-up. The use of the bunch-by-bunch electronics limits the optics perturbation to the selected bunch, only, leaving the rest of the train unaffected.

It is worthwhile stressing the fact that the islands' linear optics and hence equilibrium emittance can be adjusted via non-linear magnets, hence without altering the linear optics and beam parameters of the beam on the nominal orbit. Sextupole magnets, which are harmonic for the nominal beam, will become chromatic sextupoles (and quadrupoles) for the beam trapped in the islands: a linear system to trim these magnets while preserving the linear chromaticity for the nominal beam is then possible. Islands' chromaticity can be also adjusted via octupole magnets because of their sextupolar feed-down components.

Any further development of the above ideas requires a preliminary answer to a number of fundamental questions. Firstly, are the computing codes adapted to evaluating the properties of stable islands? Secondly, are stable islands possible at all in presence of radiation damping and diffusion? Next section addresses these two subjects.

4. Simulating electron beams dynamics into the islands

Optics studies of resonant islands at CERN were carried out via the PTC modules of MAD-X[37–40]. Several modifications to some standard modules were introduced in order to compute the linear optics around the stable fixed points. Similar modifications had to be implemented in Accelerator Toolbox (AT)[41–43], which is widely used in the synchrotron light source community. In this section we outline the main numerical ingredients for a correct description of the electron beam dynamics around the fixed points in both codes. First, multi-particle tracking results showing the possibility of having stable islands even in the case of electrons are also presented.

4.1. *Islands' linear optics and equilibrium emittance via analytic tools*

Two initial steps for the evaluation of the islands' linear optics are necessary. First, a structure (or sequence) comprising N times the base lattice, where N is the resonance order, needs to be defined, since the fixed point orbit and optics are periodic over N turns, only. Second, the optics code must be guided to search for an off-axis closed orbit: In the case of MAD-X-PTC and AT this is obtained by giving some initial conditions in phase space close to the expected fixed point. The position of the latter can be estimated by tracking some off-axis initial conditions and displaying the phase space portrait.

A third crucial ingredient is the correct evaluation of the feed-down fields experienced by the fixed points when evaluating their linear optics. If the latter is based on single-particle tracking (as in AT), this should be taken into account automatically. If truncated Hamiltonian terms are used (as in PTC), these should be properly expanded around the fixed point coordinates. If the linear optics is inferred from linear maps, these should correctly refer to the fixed points and not to the standard on-axis orbit.

For a correct evaluation of the synchrotron radiation effects on lepton beams, damping and diffusion terms need also to be properly evaluated by accounting for the non-standard dipole and quadrupole feed-down fields originated by the fixed point off-axis orbit at the location of quadrupole and sextupole magnets. If quantum diffusion is evaluated element by element, this should be automatically included. On the other hand, if one-turn diffusion matrix describing the integrated effect over the ring is used, then it must be computed from the fixed point orbit including feed-down terms. The use of the diffusion matrix is then incompatible with the simulation of the resonant trapping and the simultaneous presence of particles both in the core and in the islands, since quantum diffusion would be properly described only in one of the two regions.

The `PTC_twiss` module of MAD-X can find the closed orbit nearest to given initial conditions, expand the magnetic lattice around this orbit, and compute the corresponding linear and non-linear optics, along with all equilibrium terms (such as damping times, eigen-emittances and envelope matrix). AT can also find the fixed point orbit and has several functions for the evaluation of any beam parameters, though first tests comparing its optics and equilibrium observables revealed some inconsistencies which are still under scrutiny. Hence, the equilibrium emittance computed from multi-particle tracking has been compared to the expected values from the `PTC_twiss` module of MAD-X, only.

4.2. *Islands' equilibrium emittance via multi-particle tracking*

Tracking in MAD-X can be performed in two flavours. Radiation damping and element-by-element quantum diffusion are implemented in the native MAD-X thin-element tracking[a], whereas the more powerful thick-element tracking is carried out by a PTC module only, which, however, includes only radiation damping, but no quantum diffusion. Tracking in AT is intrinsically based on thick elements and includes both damping and diffusion, with two options for the latter: either based on a (faster) one-turn matrix element or on a (slower) element-by-element scheme. The former option has been used for all AT simulations presented here. A great advantage of AT is its possibility of parallel tracking that is highly valuable when a sizeable number of macro-particles is simulated over several tens or hundreds of thousands of turns. That is the main reason why multi-particle simulations have been run only using AT and not MAD-X. A second important reason for not using MAD-X is the absence of quantum diffusion in PTC and the need of properly slicing the lattice if the thin-lens tracking of the native MAD-X is to be used, with the coherence problems between the thick model of AT and the thin description of MAD-X that may arise. In the left plot of Fig. 5 an example of phase space portrait for the ESRF storage ring featuring three stable islands (i.e., $N = 3$) is

[a] All these features are already working for thick tracking in a development version of MAD-X and they will be part of the next official code release.

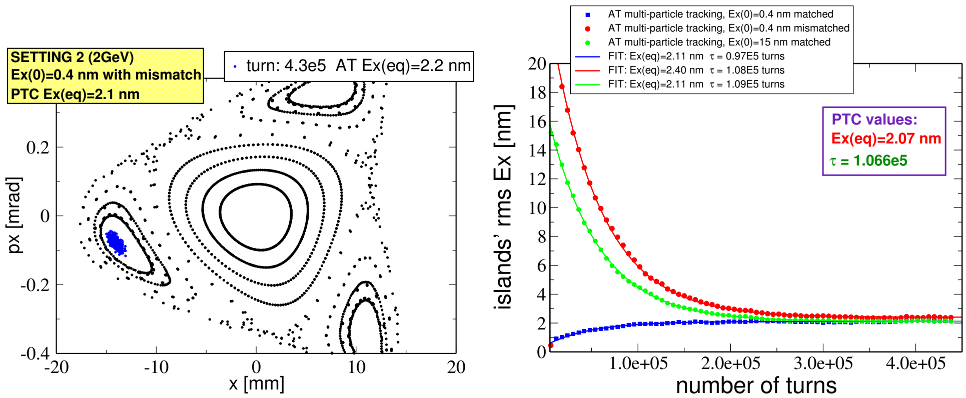

Fig. 5. Left: phase space portrait for the ESRF storage ring featuring three stable islands along with the final distribution of 10^3 macro-particles after 4.3×10^5 turns (blue dots): The initial distribution is centred around the fixed point, has an emittance of 0.4 nm (compared to 2.1 nm at equilibrium as predicted by PTC) and mismatched optical parameters. Right: rms horizontal emittance versus the number of turns. The three curves refer to different initial conditions with smaller and larger initial emittance compared to the equilibrium value predicted by PTC and with mismatched optical parameters. In all three cases we observe a convergence of the emittance towards an equilibrium. The fitted equilibrium emittance and damping time agree well with the prediction of PTC. This sextupole setting is referred to as SETTING 2.

displayed. Because of the smaller size of the linear region of the stable islands compared to that of the nominal orbit, initially, multi-particle simulations were run by lowering the beam energy to 2 GeV (instead of 6 GeV) in order to assess the existence of an equilibrium with lower natural emittance. In the right plot of Fig. 5, the rms horizontal emittance of an ensemble of 10^3 macro-particles initially placed around one fixed point of the same portrait is plotted versus time (represented by the number of turns). The three curves refer to different initial conditions with smaller and larger initial emittance compared to the equilibrium value predicted by PTC and with mismatched optical parameters. In all three cases we observe a convergence of the emittance towards an equilibrium. From the fit of the curves with the formula

$$E_x(t) = E_x(0)e^{-\frac{2t}{\tau}} + E_x(\mathrm{eq})\left[1 - e^{-\frac{2t}{\tau}}\right] . \tag{1}$$

Both the equilibrium emittance $E_x(\mathrm{eq})$, 2.2 ± 0.2 nm, and the damping time τ, $(1.05 \pm 0.07) \times 10^5$ turns, can be inferred by fitting the data. These numbers agree very well with those from PTC, 2.07 nm and 1.07×10^5 turns, respectively. The same simulations have been run by using the same lattice configuration, but increasing the beam energy so to increase the equilibrium emittance, in order to assess the existence of an equilibrium and the validity of the analytic approach of PTC when the non-linear region and contour of the islands is approached.

The left plot of Fig. 6 offers an interesting overview of the behaviour of a different

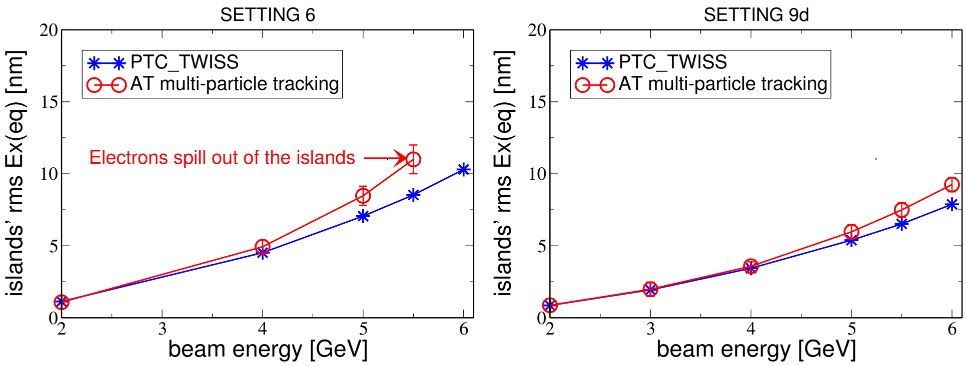

Fig. 6. Left: rms emittance computed by PTC and inferred from AT multi-particle tracking for another sextupole configuration (SETTING 6) against the beam energy. While the agreement between the two codes is good at low energies, it deteriorates around 5 GeV, while at 6 GeV multi-particle simulations indicate that all electrons moved from the island to the core. Right: same plot for yet another sextupole configuration (SETTING 9d) featuring an overall lower equilibrium emittance, better agreement between the two codes, and stable beam into the islands even at 6 GeV.

islands' configuration. At energy lower than 4 GeV the agreement between PTC and AT multi-particle tracking is good, though it deteriorates when increasing the energy up to 5.5 GeV, when a few electrons spill out the islands after 10^5 turns and move into the core. At 6 GeV, multi-particle tracking shows that after 1.2×10^7 turns all electrons left the islands, damping to the nominal orbit (see right phase space plot of Fig. 7).

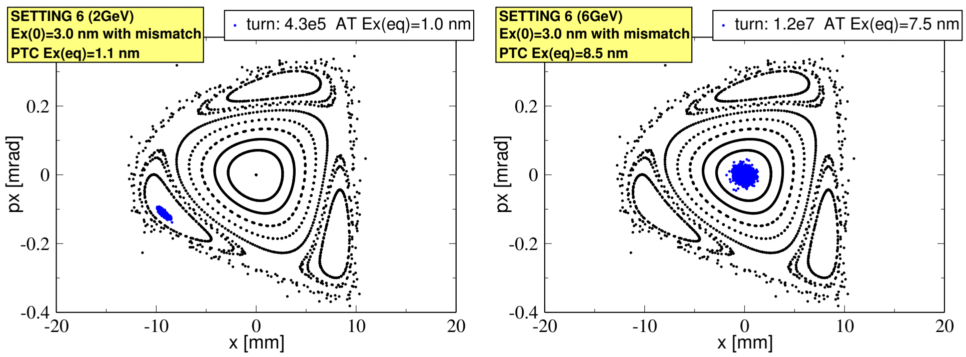

Fig. 7. Phase space portrait generated by the same non-linear optics (SETTING 6) of the left plot in Fig. 6 with superimposed final distribution at two different energies (blue dots): 2 GeV on the left, featuring stable beam trapped into the island with the same equilibrium emittance predicted by PTC, and 6 GeV on the right, where all electrons abandoned the islands to reach the core. Note that the final emittance of the beam into the core (7.5 nm) differs from the nominal ESRF value of 4 nm: This error is due to the fact that in this simulation the quantum diffusion was modelled via a one-turn matrix computed around the fixed point orbit. Such an approach does not apply once the electrons oscillate around the core. A correct description of the dynamics would have required an element-by-element approach.

Other islands' configurations (obtained by varying only the strength of the sextupole magnets) showed that stable beam into the islands can be obtained even at 6 GeV while improving the agreement between the analytic approach of PTC and the multi-particle tracking of AT, as shown in the right plot of Fig. 6. Interestingly, the difference in equilibrium emittance between the two plots of Fig. 6 results from the simple change of sextupoles, hence with no impact on the beam parameters for the nominal on-axis orbit. The reason for the larger discrepancy between the two calculations at higher energy is not well understood, yet. Two are the possible mechanisms under study. First, when the equilibrium emittance (and hence the electrons oscillating around the fixed point) approach the islands' contour, particles experience non-linear forces introducing a filamentation that adds up to the quantum diffusion, resulting hence in an effective larger diffusion not contemplated by the linear model used by the `PTC_twiss` module of MAD-X. Second, the stronger diffusion at higher energies is associated with a larger energy variation of the electrons, whose off-momentum trajectories in phase spaces may lay at the limits of, or even outside, the islands contour before the longitudinal damping restores the nominal energy. If electrons abandon the islands, either moving into the core or getting lost through the chaotic regions near the separatrix, no equilibrium is reached around the fixed point.

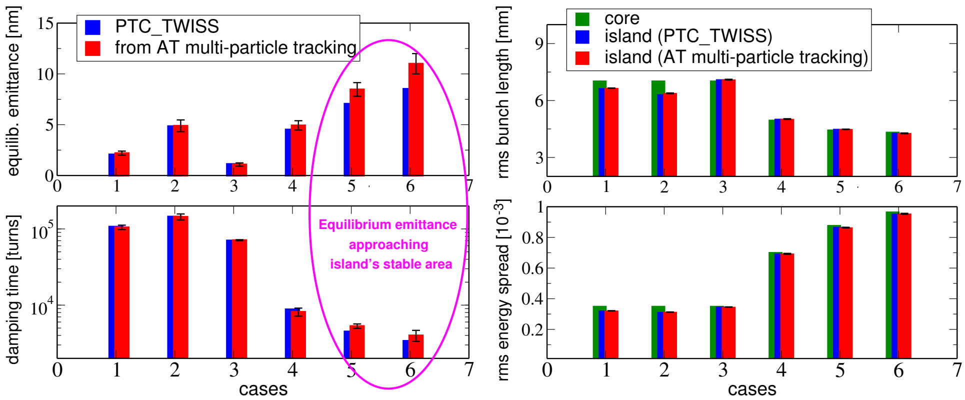

Fig. 8. Left: horizontal islands' equilibrium emittance and damping time for six different lattice configurations or beam energies as computed by `PTC_twiss` or inferred from AT multi-particle simulations. Right: core's and islands' rms bunch length and energy spread obtained by the two codes for the same six configurations.

In the left plot of Fig. 8 both horizontal equilibrium emittance and damping time for six different lattice configurations or beam energies are plotted for PTC and AT. The agreement remains good as long as the equilibrium emittance is much smaller than the islands' stable area. In the right plot of the same figure, the rms bunch length and energy spread obtained by the two codes for the same six configurations are plotted. We observe an overall better agreement in these longitudinal parameters compared to the horizontal equilibrium emittance. The values for the nominal beam into the core are also reported, showing how it is possible to have different values between islands and core: though small in these examples, these variations could be in principle enhanced by playing with non-linear magnets.

5. Conclusions and outlook

In this paper, the recent applications of stable islands for proton beams have been reviewed, discussing the dynamic and static use of stable islands in proton circular particle accelerators. The success of these applications, some of which have been implemented at CERN, while others have been studied using simple models, suggested to extend them to other domains, such as lepton beams, and, more specifically, synchrotron light rings. In this paper, we have reported about recent studies on trapping electron beams into static stable islands under the combined influence of radiation damping and quantum excitation. The resulting optical parameters and equilibrium emittances have been considered for such trapped beams, discussing the results of both semi-analytical and multi-particle tracking codes. The successful outcome of these studies suggests to pursue further these activities, in view of providing detailed analysis of the possibilities opened up by the use of stable islands for lepton beams.

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